

# Fine structure splittings of excited $P$ and $D$ states in charmonium

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## Abstract

It is shown that the fine structure splittings of the  $2^3P_J$  and  $3^3P_J$  excited states in charmonium are as large as those of the  $1^3P_J$  state if the same  $\alpha_s(\mu) \approx 0.36$  is used. The predicted mass  $M(2^3P_0) = 3.84$  GeV appears to be 120 MeV lower than the center of gravity of the  $2^3P_J$  multiplet and lies below the  $D\bar{D}^*$  threshold. Our value of  $M(2^3P_0)$  is approximately 80 MeV lower than that from the paper by Godfrey and Isgur [1] while the differences in the other masses are  $\lesssim 20$  MeV. Relativistic kinematics plays an important role in our analysis.

## 1 Introduction

At present only the  $1^3D_1$  and  $2^3D_1$  states lying above the  $D\bar{D}$  threshold have been identified with the experimentally observed  $c\bar{c}$  mesons,  $\psi(3770)$  and  $\psi(4160)$ . Still a large number of other excited  $P$ - and  $D$ -wave states above the flavor threshold were predicted. Their masses and fine structure splittings were calculated by Godfrey and Isgur (GI) already in 1985, in the framework of a relativistic approach [1]. The properties of the  $P$  and  $D$  levels in charmonium and bottomonium were also intensively studied in the nonrelativistic approximation [2, 3]. There is a point of view that one or

more charmonium  $2P_J$  states can be narrow enough to have a substantial branching ratio to the  $\gamma + \psi(2S)$  channel [4, 5] and could play a role in the hadronic production of  $\psi(2S)$  mesons. In particular, the  $2P$  states can be related to the enhancement in the  $J/\psi\pi^+\pi^-$  system near  $M = 3.84$  GeV observed in [6] (but not confirmed by another group [7]). Therefore the precise knowledge of their masses is especially important.

An accurate description of the charmonium spectrum and the fine structure splittings of the  $1P$  level was presented in our previous papers [8, 9], where, as well as in [1], relativistic kinematics was taken into account with the help of the spinless Salpeter equation. As shown in [9] the relativistic corrections to the matrix elements, like  $\langle r^{-3} \rangle$  determining spin structure, are large enough, of the order of  $\approx 40\%$ , therefore the nonrelativistic approach can not be considered to be appropriate when the spin structure is investigated.

We shall show here that in the relativistic approach the spin-orbit and tensor splittings are rather large for the  $P$  states, for the ground state as well as the excited levels. This result depends weakly on the choice of the strong coupling constant  $\alpha_s(\mu)$ . Here  $\alpha_s(\mu) \approx 0.36$  ( $\mu = 0.92$  GeV) will be used for all states, but the splittings do practically not change if  $\alpha_s(\mu) = 0.30$  ( $\mu = m = 1.48$  GeV) is taken.

The fine structure splittings predicted here appear to be larger than those in GI's paper [1], especially for the  $2P$  and  $3P$  states. The reasons for that will be discussed in Section 3. As a result the  $2^3P_0$  mass,  $M(2^3P_0) = 3.84$  GeV in our case appears to be approximately 80 MeV smaller than in [1] and this level lies below the  $D\bar{D}^*$  threshold. The  $2^3P_{1,2}$  levels as well as the  $n^3D_J$  states have mass values close to the GI predictions. For the first time we also predict large fine structure splittings for the  $3P$  states.

## 2 Spin-averaged spectrum

The relativistic effects in charmonium are expected not to be small, especially for the wave functions and matrix elements which we are mostly interested in here. Therefore to find the spin-averaged spectrum the spinless Salpeter equation will be solved as already done in several papers [1, 10, 11]

$$\left(2\sqrt{p^2 + m^2} + V_0(r)\right) \psi_{nl}(r) = M_{nl} \psi_{nl}(r). \quad (1)$$

The static interaction  $V_0(r)$  will be taken in the form of the Cornell potential,

$$V_0(r) = -\frac{4}{3}\frac{\tilde{\alpha}}{r} + \sigma r + C_0 \quad (2)$$

and the values of  $\tilde{\alpha} \equiv \alpha_V(\mu)$ , the string tension  $\sigma$  and the pole mass of the  $c$  quark will be taken as in our previous paper [8] while fitting the fine structure of the  $1P$  charmonium state,

$$m = 1.48 \text{ GeV}, \quad \sigma = 0.18 \text{ GeV}^2, \quad \tilde{\alpha} = 0.42. \quad (3)$$

The constant  $C_0$  in Eq. (2) is determined from a fit to the spin-averaged mass of the  $1S$  states,  $\overline{M}(1S) = 3067.6 \text{ MeV}$  [12], and from there  $C_0 = -140.2 \text{ MeV}$ . In our approach the strong coupling constant  $\tilde{\alpha}$  is not an independent parameter. It can be connected to  $\alpha_s(\mu)$  (in the  $\overline{MS}$  renormalization scheme) through the relation [13]

$$\tilde{\alpha}(q^2) = \alpha_V(q^2) = \alpha_s(\mu) \left( 1 + \frac{1.75}{\pi} \alpha_s(\mu) \right) \quad (n_f = 3). \quad (4)$$

For our choice of  $\tilde{\alpha}$  and  $\alpha_s(\mu)$  (see below) this relation will be valid with an accuracy better than 5%.

The chosen parameters (3) can be compared to the GI parameters in [1] where  $m = 1.628 \text{ GeV}$  while  $\sigma = 0.18 \text{ GeV}^2$  coincides with  $\sigma$  in (3). In [1] a running coupling constant was used for  $\tilde{\alpha}$  with the critical value  $\alpha_{cr} = \tilde{\alpha}(r = 0) = 0.60$  which is larger than the constant  $\tilde{\alpha} = 0.42$  in our case. Also in [1]  $C_0 = -253 \text{ MeV}$  whereas  $C_0 = -140 \text{ MeV}$  in our calculations. Nevertheless, the calculated spin-averaged masses for both sets of parameters are close to each other. For  $P$ -wave states the differences are less than 10 MeV and for  $D$ -wave states less than 20 MeV (see Table 1).

In many papers excited states of charmonium were studied in the non-relativistic approximation which works quite well for the spectrum. But it was shown in [8, 9] that the relativistic corrections to matrix elements like  $\langle r^{-3} \rangle$  and  $\langle r^{-3} \ln mr \rangle$ , which determine fine structure splittings, are rather large, about 30 ÷ 40%. That is why in this paper only relativistic calculations of the fine structure splittings of the excited states of charmonium will be considered.

### 3 Fine structure parameters of the $P$ levels

Although the spin-averaged masses in our calculations are very close to those in GI's paper [1], we expect that the spin-orbit and tensor splittings of the  $P$ -wave states will be larger in our case. There are two reasons for that. First, we take into account the corrections to second order in  $\alpha_s$ . Secondly, our calculations of different matrix elements have shown that for the excited  $P$  states the matrix element  $\langle r^{-3} \rangle$  which determines the splittings, is not decreasing. For the set of parameters (3) it was found that in the relativistic case  $\langle r^{-3} \rangle_{1P} = 0.142$ ,  $\langle r^{-3} \rangle_{2P} = 0.157$ ,  $\langle r^{-3} \rangle_{3P} = 0.167$ , i.e.  $\langle r^{-3} \rangle_{nP}$  are even increasing for  $2P$  and  $3P$  states. This result is specific to the Salpeter equation, whereas in the nonrelativistic case the matrix elements  $\langle r^{-3} \rangle$  for the excited states are decreasing, e.g.  $\langle r^{-3} \rangle_{1P} = 0.101$ ,  $\langle r^{-3} \rangle_{2P} = 0.093$ ,  $\langle r^{-3} \rangle_{3P} = 0.089$ . The accuracy of our calculations was checked to be  $(1 \div 2)10^{-4}$ .

The fine structure parameters are defined as matrix elements of the spin-orbit and tensor interactions,

$$a = \langle \tilde{V}_{LS}(r) \rangle, \quad c = \langle \tilde{V}_T(r) \rangle, \quad (5)$$

where the scalar functions  $\tilde{V}_{LS}(r)$  and  $\tilde{V}_T(r)$  are introduced as

$$\begin{aligned} \hat{V}_{LS}(r) &= \tilde{V}_{LS}(r) \vec{L} \cdot \vec{S}, \\ \hat{V}_T(r) &= \tilde{V}_T(r) \hat{S}_{12}, \quad \hat{S}_{12} = 3(\vec{s}_1 \cdot \vec{n})(\vec{s}_2 \cdot \vec{n}) - \vec{s}_1 \cdot \vec{s}_2, \quad \vec{n} = \frac{\vec{r}}{r}. \end{aligned} \quad (6)$$

Here the spin-orbit parameter  $a$  is defined in the same manner as in other papers, whereas the definition of the tensor parameter  $c$  differs from [1] where the tensor parameter  $T = \frac{1}{2}c$  and from [2] where the parameter  $b$  is used, related to  $c$  by  $b = 4c$ .

In our calculations we suggest that the  $P$ -wave hyperfine splitting is small as it occurs for the  $h_c$  ( $1P$ ) meson for which the hyperfine shift relative to the center of gravity of the  $^3P_J$  multiplet,  $\overline{M}(1^3P_J)$ , is less than 1 MeV. When the hyperfine splitting is neglected the mass of the states with spin  $S = 0$  coincides with the center of gravity of the  $^3L_J$  multiplet denoted by  $\overline{M}_L$ . Their values, taken from Table 1, are

$$\begin{aligned} M(1^1P_1) &= 3528 \text{ MeV}; M(2^1P_1) = 3962 \text{ MeV} \\ M(1^1D_2) &= 3822 \text{ MeV}; M(2^1D_2) = 4194 \text{ MeV}, \end{aligned} \quad (7)$$

which are about 20 MeV lower than in [1] for the  $D$  and some  $S$  states.

For the states with spin  $S = 1$  and orbital angular momentum  $L \neq 0$  the mass of the state can be represented as

$$M(^3L_J) = \overline{M}_L + a < \vec{L} \cdot \vec{S} > + c < \hat{S}_{12} > \quad (8)$$

with the operator  $\hat{S}_{12}$  defined as in (6). For  $P$ -wave states it gives

$$M(^3P_2) = \overline{M}_1 + a - \frac{1}{10}c, \quad M(^3P_1) = \overline{M}_1 - a + \frac{1}{2}c, \quad M(^3P_0) = \overline{M}_1 - 2a - c. \quad (9)$$

In Ref. [1] only the terms of first order in  $\alpha_s$  were taken into account and for  $a$  and  $c$  the following values were obtained,

$$\begin{aligned} a(1P) &= 28 \text{ MeV}, & c(1P) &= 26 \text{ MeV}, \\ a(2P) &\approx 17 \text{ MeV}, & c(2P) &\approx 8 \text{ MeV}, \end{aligned} \quad (10)$$

which are about 20% and 30% resp. smaller for the  $1P$  state compared to the current experimental values [8]:

$$a_{\text{exp}}(1P) = 34.56 \pm 0.19 \text{ MeV} \quad c_{\text{exp}}(1P) = 39.12 \pm 0.62 \text{ MeV}. \quad (11)$$

In our approach the terms of second order in  $\alpha_s$  will be taken into account and the total value of  $a$  and  $c$  can be represented as

$$a_{\text{tot}} = a_P^{(1)} + a_P^{(2)} + a_{\text{NP}}, \quad c_{\text{tot}} = c_P^{(1)} + c_P^{(2)} + c_{\text{NP}}, \quad (12)$$

where the nonperturbative contribution to the spin-orbit splitting coming from the linear confining potential is

$$a_{\text{NP}} = -\frac{\sigma}{2m^2} < r^{-1} >. \quad (13)$$

In the tensor splitting, Eq. (12), the small nonperturbative term  $c_{\text{NP}}$  will be neglected, (see the discussion in [9]). Godfrey and Isgur took into account non-perturbative spin effects. In order to do so, they needed to smear their potentials at short range, which makes it necessary to introduce additional unknown parameters. Here we consider spin-effects as a perturbation using explicit analytical expressions for the spin-orbit and tensor potentials in

coordinate space in the  $\overline{MS}$  renormalization scheme from [15]. The terms to first order in  $\alpha_s$  are  $a_P^{(1)}$  and  $c_P^{(1)}$  given by

$$a_P^{(1)} = \frac{2\alpha_s(\mu)}{m^2} \langle r^{-3} \rangle, \quad c_P^{(1)} = \frac{4}{3} \frac{\alpha_s(\mu)}{m^2} \quad (14)$$

and the second order perturbative corrections are

$$\begin{aligned} a_P^{(2)} &= \frac{2\alpha_s^2(\mu)}{\pi m^2} \left\{ 4.5 \ln \frac{\mu}{m} \langle r^{-3} \rangle + 2.5 \langle r^{-3} \ln(mr) \rangle + 1.582 \langle r^{-3} \rangle \right\} \\ c_P^{(2)} &= \frac{4\alpha_s^2(\mu)}{3\pi m^2} \{ 4.5 \ln \frac{\mu}{m} \langle r^{-3} \rangle + 1.5 \langle r^{-3} \ln(mr) \rangle + 3.449 \langle r^{-3} \rangle \} \end{aligned} \quad (15)$$

The second order expressions are given here for a number of flavors  $n_f = 3$ . (We checked that for  $n_f = 4$  the values of  $a$  and  $c$  do not practically change—the differences are less than 0.5 MeV—therefore only the case  $n_f = 3$  will be presented here.)

With the solutions of the Salpeter equation (1) all matrix elements determining Eqs (13-15) can be calculated and the only uncertainty comes from the choice of the strong coupling constant  $\alpha_s(\mu)$  and the value of the renormalization scale  $\mu$ . In [9] it was found that for the  $1P$  state in charmonium

$$\alpha_s(\mu) = 0.365 \quad (\mu = 0.92 \text{ GeV}) \quad (16)$$

gives an accurate description of the spin splittings. For the excited  $2P$  and  $3P$  states where experimental data are absent, we shall use the same value (16) for  $\alpha_s(\mu)$ . The main argument in favour of this choice can be taken from the fine structure analysis in bottomonium where the values of  $\alpha_s(\mu)$  for the  $1P$  and  $2P$  states differ by about 10% only [8].

With  $\alpha_s(\mu) = 0.365$  the authors of [14] obtained for the  $1P$  state

$$\begin{aligned} a_P^{(1)}(1P) &= 47.6 \text{ MeV}, \quad a_P^{(2)}(1P) = 3.6 \text{ MeV}, \quad a_{\text{NP}}(1P) = -16.6 \text{ MeV}, \\ c_P^{(1)}(1P) &= 31.7 \text{ MeV}, \quad c_P^{(2)}(1P) = 7.4 \text{ MeV}, \end{aligned} \quad (17)$$

so that  $a_{\text{tot}}(1P)$  and  $c_{\text{tot}}(1P)$  just coincide with their experimental values (11).

For the excited  $2P$  state the spin-orbit and tensor parameters are as large as those of the  $1P$  state, because the matrix element  $\langle r^{-3} \rangle_{2P}$  is even  $\approx 10\%$  larger than  $\langle r^{-3} \rangle_{1P}$ . Here we face the difference between the

relativistic and nonrelativistic approaches. For the latter, matrix elements like  $\langle r^{-3} \rangle_{nP}$  are decreasing with growing  $n = n_r + 1$ . Our calculations give for the  $2P$  state that

$$\begin{aligned} a_P^{(1)}(2P) &= 52.5 \text{ MeV}, & a_P^{(2)}(2P) &= -0.4 \text{ MeV}, & a_{\text{NP}}(2P) &= -13.4 \text{ MeV}, \\ c_P^{(1)}(2P) &= 35.0 \text{ MeV}, & c_P^{(2)}(2P) &= 6.5 \text{ MeV} \end{aligned} \quad (18)$$

so that

$$a_{\text{tot}}(2P) = 38.7 \text{ MeV}, \quad c_{\text{tot}}(2P) = 41.5 \text{ MeV} \quad (19)$$

are even slightly larger than the corresponding values for the  $1P$  state.

Comparing values obtained for  $a(2P)$  and  $c(2P)$  with those in (11) from GI's paper one can see that  $a$  and  $c$  in our calculations are larger by a factor of 2 or 5 resp. than the values from GI's paper (see also Table 2). This discrepancy is partly connected to our taking into account of the second order radiative corrections which are, however, not large. But even with only first order perturbative terms included, our values of  $a(2P)$  and  $c(2P)$  are much larger than GI's values (11).

Taking  $a$  and  $c$  from Eq. (19) and  $\overline{M}_1(2P) = 3962 \text{ MeV}$  (from Table 1) one obtains the following masses for the  $2^3P_J$  states,

$$M(2^3P_0) = 3843 \text{ MeV}, \quad M(2^3P_1) = 3944 \text{ MeV}, \quad M(2^3P_2) = 3997 \text{ MeV}. \quad (20)$$

Our predicted mass for the  $2^3P_0$  state,  $M(2^3P_0) = 3.84 \text{ GeV}$  appeared to be  $\approx 80 \text{ MeV}$  lower than the one in GI's paper, whereas for the other two states,  $2^3P_1$  and  $2^3P_2$ , the predicted masses only slightly differ from the GI values (see Table 3) due to a cancelation of terms with different signs.

It is important that in our calculation the  $2^3P_0$  level lies below the  $D\bar{D}^*$  threshold ( $M_{\text{th}}(D\bar{D}^*) \approx 3.87 \text{ GeV}$ ) but higher than  $M_{\text{th}}(D\bar{D}) = 3.73 \text{ GeV}$ . This fact can affect the decay rates of the  $2^3P_0$  state.

For the  $3P$  state, using again  $\alpha_s(\mu) = 0.365$  and  $\mu = 0.92 \text{ GeV}$  as for the  $1P$  state, one obtains using Eqs. (13-15)

$$\begin{aligned} a_P^{(1)}(3P) &= 56.7 \text{ MeV}, & a_P^{(2)}(3P) &= -3.6 \text{ MeV}, & a_{\text{NP}}(3P) &= -11.8 \text{ MeV}, \\ c_P^{(1)}(3P) &= 37.8 \text{ MeV}, & c_P^{(2)}(3P) &= 6.2 \text{ MeV}, & (c_{\text{NP}} &= 0), \end{aligned} \quad (21)$$

so that

$$a_{\text{tot}}(3P) = 42.3 \text{ MeV}, \quad c_{\text{tot}}(3P) = 44.0 \text{ MeV}. \quad (22)$$

With these values for  $a$  and  $c$  and the spin-averaged mass  $\overline{M}_1(3P) = 4320$  MeV it follows that

$$M(3^3P_1) = 4300 \text{ MeV}, \quad M(3^3P_2) = 4358 \text{ MeV}, \quad M(3^3P_0) = 4192 \text{ MeV}. \quad (23)$$

The level  $3^3P_0$  lies 128 MeV lower than the center of gravity of the  $3^3P_J$  multiplet. It is of interest to notice that the difference  $M(n^3P_2) - M(n^3P_0) \equiv \Delta(nP) = 3a + 0.9c$  is large in all cases and slightly increasing for the excited states,

$$\Delta(1P) = 138.9 \text{ MeV}, \quad \Delta(2P) = 143 \text{ MeV}, \quad \Delta(3P) = 166 \text{ MeV}. \quad (24)$$

We have checked the sensitivity of the predicted values for  $a$  and  $c$  to the choice of the renormalization scale  $\mu$  and  $\alpha_s(\mu)$ . To this end the commonly used value of  $\mu = m$  and  $\alpha_s(\mu = m = 1.48 \text{ GeV}) = 0.29$  was considered. Then for the  $2P$  state  $a_{\text{tot}}(2P) = 36.5 \text{ MeV}$  and  $c_{\text{tot}}(2P) = 37.5 \text{ MeV}$  and for the  $3P$  state  $a_{\text{tot}}(3P) = 40.5 \text{ MeV}$  and  $c_{\text{tot}}(3P) = 39.8 \text{ MeV}$  were obtained, which are very close to the values (20), (21), (23), and (24) with  $\mu_0 = 0.92 \text{ GeV}$  and  $\alpha_s(\mu_0) = 0.365$ , found in [8] from the fit to the  $1P$  fine structure splittings.

## 4 Fine structure splittings of the $D$ levels

For the  $D$  state the expressions of the splitted masses,  $M(n^3D_J)$ , through the parameters  $a$  and  $c$  can be found in [2],

$$M(^3D_1) = \overline{M}_2 - 3a - \frac{1}{2}c, \quad M(^3D_2) = \overline{M}_2 - a + \frac{1}{2}c, \quad M(^3D_3) = \overline{M}_2 + 2a - \frac{1}{7}c. \quad (25)$$

For the spin-averaged masses  $\overline{M}(nD)$  our calculation with the parameters (3) give

$$\overline{M}_2(1D) = 3822 \text{ MeV}, \quad \overline{M}_2(2D) = 4194 \text{ MeV}. \quad (26)$$

The fine structure parameters  $a$  and  $c$  for the  $D$ -wave levels are given in Table 2 together with their values from GI's paper. As seen from Table 2,  $a$  and  $c$  in both cases practically coincide. Still our predicted masses for the  $1^3D_J$  and  $2^3D_J$  states appear to be about 20 MeV lower than in [1] because of the smaller value of the spin-averaged masses.



## 5 Conclusion

In our analysis it was found that

- (i) in the relativistic case the fine structure splittings of the charmonium  $P$ -states with spin  $S = 1$  are much larger than in the nonrelativistic case;
- (ii) for the excited  $2P$  states the parameters of the fine structure are even slightly larger than for the  $1^3P_J$  ground state;
- (iii) the mass of the  $n^3P_0$  states ( $n = 1, 2, 3$ ) appears to be about 130 MeV smaller than the center of gravity of the  $n^3P_J$  multiplet. This fact can be important for the explanation of the decays of the excited states of charmonium.

The value we predict for  $M(2^3P_0) = 3.84$  GeV is about 80 MeV lower than in GI's calculations [1]. This state lies below the  $D\bar{D}^*$  threshold and is only about 100 MeV higher than the  $D\bar{D}$  threshold. The point of view exists that this state could be very broad because it lies above the  $D\bar{D}$  threshold and therefore should have a large hadronic width. On the other hand this state lies relatively close to the  $D\bar{D}$  threshold, so the width could be suppressed by phase space limitations. Therefore this state could play a role in the production of  $\psi(2S)$  charmonium mesons as it was discussed in [5, 6].

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Table 1: The spin-average masses  $M(nL)$  (in MeV) in charmonium for two sets of parameters.

$M(nL)$	Godfrey and Isgur $m = 1.628\text{GeV}$ $\sigma = 0.18\text{GeV}^2$ $\tilde{\alpha}_{crit} = 0.6$ (running $\tilde{\alpha}(r)$ ) $C_0 = -253$ MeV	present paper $m = 1.48$ GeV $\sigma = 0.18\text{GeV}^2$ $\tilde{\alpha} = 0.42$ , $C_0 = -140$ MeV	Experiment
$\psi(1S)$	3067.5	3067.6	$3067.6 \pm 0.6$
$\psi(2S)$	3665	$3659^a)$	$3663 \pm 1.3$
$\psi(3S)$	4090	$4077^b)$	$4040 \pm 10$
$\psi(4S)$	4450	4425	$4415 \pm 6$
$\psi(5S)$		4732	
$\chi_c(1P)$	3520	3528	$3525.5 \pm 0.4$
$\chi_c(2P)$	3960	3962	
$\chi_c(3P)$		4320	
$M(1D)$	3840	3822	$3768.9 \pm 2.5$
$M(2D)$	4210	4194	$4159 \pm 20$
$M(3D)$	4520	4519.5	

<sup>a)</sup> Mixing of  $2S$  and  $1D$ -wave states is not taken into account

<sup>b)</sup> Mixing of  $3S$  and  $2D$ -wave states is not taken into account

Table 2: Spin-orbit and tensor splittings  $a$  and  $c$  (in MeV) for  $P$  and  $D$  levels.

	Godfrey and Isgur paper <sup>a)</sup>	present paper $\alpha_s(\mu) = 0.365$	Experiment
$a(1P)$	28	34.56	$34.56 \pm 0.19$
$c(2P)$	26	39.12	$39.12 \pm 0.62$
$a(2P)$	17	38.7	
$c(2P)$	8	41.5	
$a(3P)$		42.3	
$c(3P)$		44.0	
$a(1D)$	$\approx 5$	3.64	
$c(1D)$	$\approx 10$	10.94	
$a(2D)$	$\approx 5$	5.43	
$c(2D)$	$\approx 10$	11.37	

<sup>a)</sup> the values of  $a, c$  for  $2P$  and  $D$  states are extracted from the masses  $M(^3D_J)$  and  $M(2^3P_J)$  given in Ref. [1].

Table 3: Masses of the  $n^3P_J$ , and  $n^3D_J$  states (in MeV) in charmonium.

	Godfrey and Isgur paper	present paper $\alpha_s(\mu) = 0.365$	Experiment
$2^3P_0$	3920	3843	
$2^3P_1$	3950	3944	
$2^3P_2$	3980	3996	
$3^3P_0$		4192	
$3^3P_1$		4300	
$3^3P_2$		4358	
$1^3D_1$	3820	$3800^a)$	$3768.9 \pm 2.5$
$1^3D_2$	3840	3823	
$1^3D_3$	3850	3827	
$2^3D_1$	4190	$4167^b)$	$4159 \pm 20$
$2^3D_2$	4210	4195	
$2^3D_3$	4220	4204	

<sup>a)</sup> see footnote <sup>a)</sup> to Table 1; <sup>b)</sup> see footnote <sup>b)</sup> to Table 1.